Q.P. Code: 19HS0830 **R19**

Reg. No: | | | | | | | | | | |

SIDDHARTH INSTITUTE OF ENGINEERING & TECHNOLOGY:: PUTTUR

(AUTONOMOUS)

B.TECH I Year I Semester Regular Examinations Jan-2020

ALGEBRA AND CALCULUS

(Common to all Branches)

Time: 3 hours Max. Marks: 60

(Answer all Five Units **5 x 12 = 60** Marks)

UNIT-I

1 Reduce the Quadratic form $3x_1^2+3x_2^2+3x_3^2+2x_1x_2+2x_1x_3-2x_2x_3$ into canonical form by Orthogonal transformation and Find the Rank, Index and Signature of the canonical form. **12M**

OR

2 **a** If the following system has a non-trivial solution, prove that $a+b+c=0$ or $a=b=c$. $ax+by+cz=0, bx+cy+az=0, cx+ay+bz=0.$ **6M**

b If $A=\begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$, then find A^{-2} and A^{-3}. **6M**

UNIT-II

3 **a** Verify Rolle's theorem for $f(x)=(x-a)^m(x-b)^n$ in $(a, b), m>0, n>0$. **6M**

b Apply Taylor's theorem, prove that $\log_e(1+x)<1-\frac{x^2}{2}+\frac{x^3}{3}$ whenever $x>0$. **6M**

OR

4 **a** Verify Cauchy's mean value theorem for f(x) = e^x and g(x) = e^{-x} in [a, b]. **6M**

b Using Maclaurin's series expand $\tan x$ up to the fifth power of x and hence find the series for log (sec x). **6M**

UNIT-III

5 **a** Find the shortest and longest distance from the point $(3,1,-1)$ *to the sphere* $x^2+y^2+z^2=4$. **6M**

b If $u=x\sqrt{(1-y^2)}+y\sqrt{(1-x^2)}$ *and* $v=sin^{-1}x+sin^{-1}y$, *then* show that u, v are functionally dependent. **6M**

OR

6 Show that the diameter of the right circular cylinder of greatest curved surface which can be inscribed in a given cone is equal to the radius of the cone. **12M**

UNIT-IV

7 Evaluate $\iiint \frac{dxdydz}{(x+y+z+1)^3}$ over the region bounded by the coordinate planes and the plane $x+y+z=1$. **12M**

OR

8 a Evaluate the integral by changing the order of integration $\int_0^{\infty}\int_x^{\infty}\frac{e^{-y}}{y}dydx$. **6M**

b Evaluate $\int\int(x^2+y^2)dxdy$ in the positive quadrant for which $x+y\le 1$. **6M**

UNIT-V

9 a Prove that $\int_0^1(\log\frac{1}{x})_{n-1}\ dx=\tau(n)$. **6M**

b Find the values of $\Gamma\left(-\frac{7}{2}\right)$ and $\Gamma\left(-\frac{1}{2}\right)$. **6M**

OR

10 a Prove that $\int_0^1\frac{x}{\sqrt{1-x^5}}\ dx=\frac{1}{5}B\left(\frac{2}{5},\frac{1}{2}\right)$. **6M**

b Evaluate $\int_0^1 x^4\left(\log\frac{1}{x}\right)^3 dx$ **6M**

*** END ***